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STABILITY APPROACH TO TESTING THE CONSTITUTIVE RELATIONS OF SUPERPLASTICITY

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An approach to testing the suitability of the constitutive relations for describing superplasticity is proposed based on an analysis of the stability conditions of uniform extension of samples with respect to small perturbations of the current configuration. The necessity of accounting for the partial preservation of the local topology in varying the equations of motion is shown using a simple (nonlinear viscous) model of a superplastic medium.

Key words: *superplasticity, stability, methods of describing motion.*

Formulation of the Problem. At present, the superplasticity phenomenon — the development of abnormally large irreversible strains at small loads — has been found for many polycrystalline materials of definite structure in standard tests (for example, in uniaxial tension) [1]. The complexity of this phenomenon makes it important to define and classify this effect in rational exact terms of theoretical physics in a broad sense and to develop procedures for testing the suitability of the constitutive relations describing superplastic deformation modes. The present paper deals with the well-known but insufficiently developed approach [2, 3] based on the characteristic property of superplastic deformation modes — the stability of the current body configuration with respect to small or finite perturbations. Below, we study uniaxial tension because of the simplicity of this process and because of the significant effect of free-surface perturbations on the behavior of the sample. An analysis is made of the stability of uniform deformation with respect to small perturbations. This yields "conservative" superplasticity conditions which do not rule out stable nonuniform deformation modes.

To formulate the flow stability problem for a superplastic medium, whose rheology is dominated by a nonlinear viscous (with infinitesimal memory) component [1], it is necessary to analyze existing approaches. For this problem, the approach based on the Eulerian method of describing motion seems to be the most suitable for media that do not possess memory (or have infinitesimal memory) yields results inconsistent to experimental data. At the same time, the Lagrangian approach, which is usually employed to describe the behavior of materials with slowly decaying memory, provides calculation results in good agreement with experimental data. It has been established that the superplasticity mode is due (in a very narrow range of parameters) to the abnormally high sensitivity to the strain rate (in uniaxial macrouniform tension tests, the degree of sensitivity is determined by the parameter $m = d\ln\sigma/d\ln\xi$, where σ and ξ are the tensile stress and the rate deformation). For high-temperature creep of metals, $m = 1/40$–$1/3$, whereas in the case of superplasticity, it can reach values approximately equal to unity.

We consider the following example. At the current time, let a rectangular sample in the shape of a strip of length l and width b be extended uniformly at a rate $\dot{l}$. In addition, the sample is incompressible, i.e., $\xi = \dot{l}/l = -\dot{b}/b > 0$. The medium is considered nonlinear viscous. Examining uniform perturbations of the configuration and rate field of the sample at the given time at a constant force P and denoting the difference of the fields in the perturbed and main motions by δ, we obtain $\delta P = b\,\delta\sigma + \sigma\,\delta b = 0$, $\delta\ln\sigma = m\,\delta\ln\xi$, and $\delta\xi = -\delta\dot{b}/b + \dot{b}\,\delta b/b^2$ if ξ is understood in the Lagrangian sense, or $\delta\xi = -\delta\dot{b}/b$ if ξ is understood in the Eulerian sense. This implies the equation

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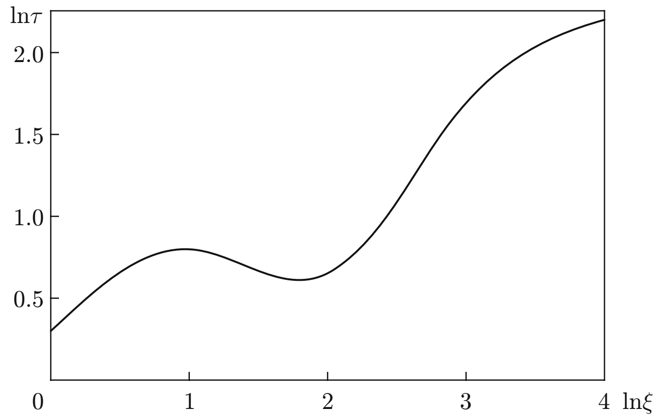

Fig. 1. Dependence $\ln\tau(\ln\xi)$.

$\delta\dot{b}/\delta b = \lambda\xi$, where $\lambda = (1-m)/m$ or $\lambda = 1/m$, respectively. The solution of this equation, $\delta b = \delta b_0 \exp(\lambda\xi t)$, indicates that in the Lagrangian description, the perturbation δb_0 decays for $m > 1$ (the Hart criterion) and $m < 0$, and in the Eulerian description, it decays only for $m < 0$. Ignoring the case of rate softening $m < 0$, the discrepancy between the result of the Eulerian stability analysis and experimental data becomes obvious.

In three-dimensional space, the situation is similar: if the problem is formulated in the current Lagrangian state, then, in particular, the variation of the rate gradient $\delta(\boldsymbol{v}\nabla) = (\delta\boldsymbol{v})\nabla - \boldsymbol{v}(\delta\nabla)$ (and refers to a material point), and in the Eulerian state, $\delta(\boldsymbol{v}\tilde{\nabla}) = (\delta\boldsymbol{v})\tilde{\nabla}$ (and refers to the point in space occupied by the material point considered). In the first case, the nabla vector ∇ is considered material and its variation characterizes the perturbation of the configuration. In the Eulerian approach, it is not possible to perturb the spatial nabla vector $\tilde{\nabla}$, and, as a result, an important term is lost from the operator and boundary conditions of the perturbed problem.

The stability of tension of a strip of a linear viscous ideally plastic material with respect to arbitrary small perturbations of the boundary in the Eulerian descriptions was studied in the classical papers [4, 5]. Shishkina et al., [6] solved a similar problem for a nonlinear viscous material. Hart [2], using the Lagrangian approach for the superplasticity mode, obtained a stability condition for tension of a strip of a nonlinear viscous material with respect to a uniform perturbation. Klyushnikov [7] pointed out the problem of choosing a method for describing motion in the stability analysis of liquid flows. The problem consists of constructing the constitutive relations of motion for media that occupy an intermediate position between solids and liquids and exhibit local topology changes (changes in neighbor particles). In the present paper, we propose an intermediate method of varying the equations, which takes into account the continually changing local topology, corresponds to the Lagrangian or Eulerian descriptions in the limiting cases, and is probably more suitable for describing stability problems for the examined class of media.

Perturbed Equations of Motion for a Nonlinear Viscous Medium. As a simple model for a superplastic medium we use the tensor-linear nonlinear viscous constitutive equations assuming incompressibility of the material. For this medium, the equilibrium equations for rates in terms of the Lagrangian current approach are written as

$$\nabla\cdot\sigma = \mathbf{0}, \qquad \sigma = -pI + \frac{\tau(\xi)}{\xi}\,D,$$

$$D = (\nabla\boldsymbol{v} + \boldsymbol{v}\nabla)/2, \qquad \nabla\cdot\boldsymbol{v} = 0, \tag{1}$$

where ∇ is the material gradient operator in the current configuration, σ and D are the stress and strain rate tensors, p is the hydrostatic pressure, I is the second rank unit tensor, $\tau(\xi)$ is the material function, $\xi = \sqrt{(1/2)D : D}$, and $\boldsymbol{v}$ is the rate of displacement. The dependence $\ln\tau(\ln\xi)$ is assumed to be sigmoid, typical of a medium that exhibits superplasticity, probably with a segment on which the rate softening decreases (Fig. 1). In the definition of ξ, the factor is chosen so that it can could be used in the two-dimensional problem studied below, for which $\tau = \sqrt{2\sigma' : \sigma'}$ (the prime denotes the deviator part).

Perturbing (1), we obtain

$$\nabla \cdot \delta\sigma + \delta\nabla \cdot \sigma = \mathbf{0}, \qquad \delta\sigma = -\delta p I + \Big(\frac{\tau'}{\xi} - \frac{\tau}{\xi^2}\Big)\delta\xi D + \frac{\tau}{\xi}\,\delta D,$$
$$\delta D = (\nabla\delta\boldsymbol{v} + \delta\boldsymbol{v}\nabla)/2 + ((\delta\nabla)\boldsymbol{v} + \boldsymbol{v}(\delta\nabla))/2, \qquad \nabla \cdot \delta\boldsymbol{v} + \delta\nabla \cdot \boldsymbol{v} = 0. \tag{2}$$

In view of the relations [8]

$$\delta\nabla = -(\nabla\delta\boldsymbol{u}) \cdot \nabla, \qquad \delta\xi = \frac{1}{2\xi}\,D : \delta D, \qquad \frac{\tau'}{\tau}\,\xi = \frac{d\ln\tau}{d\ln\xi} \equiv m,$$

system (2) becomes

$$\nabla \cdot \delta\sigma - \nabla\delta\boldsymbol{u} : \nabla\sigma = \mathbf{0}, \qquad \delta\sigma = -\delta p I + \frac{\tau}{\xi}\Big(\frac{m-1}{2}\,\frac{D}{\xi}\,\frac{D}{\xi} + I_4\Big) : \{\delta\boldsymbol{v}\nabla - \boldsymbol{v}\nabla \cdot \delta\boldsymbol{u}\nabla\},$$
$$\nabla \cdot \delta\boldsymbol{v} - \boldsymbol{v}\nabla : \delta\boldsymbol{u}\nabla = 0, \tag{3}$$

where $A : B = \operatorname{tr} A \cdot B$; I_4 is unity in the space of fourth rank tensors and $\{A\} = (A + A^{\rm t})/2$. It should be taken into account that

$$\delta\boldsymbol{v} = \delta\dot{\boldsymbol{u}}. \tag{4}$$

In the Eulerian approach, the operator ∇ is assumed to be a spatial one and the perturbed system (1) is easily obtained from (3) ignoring the terms containing the perturbation of the current configuration $\delta\boldsymbol{u}$:

$$\nabla \cdot \delta\sigma = \mathbf{0}, \qquad \delta\sigma = -\delta p I + \frac{\tau}{\xi}\Big(\frac{m-1}{2}\,\frac{D}{\xi}\,\frac{D}{\xi} + I_4\Big) : \{\delta\boldsymbol{v}\nabla\},$$
$$\nabla \cdot \delta\boldsymbol{v} = 0. \tag{5}$$

Along with (3) and (5), it is proposed to consider the intermediate formulation

$$\nabla \cdot \delta\sigma - \chi\nabla\delta\boldsymbol{u} : \nabla\sigma = \mathbf{0}, \qquad \delta\sigma = -\delta p I + \frac{\tau}{\xi}\Big(\frac{m-1}{2}\,\frac{D}{\xi}\,\frac{D}{\xi} + I_4\Big) : \{\delta\boldsymbol{v}\nabla - \chi\boldsymbol{v}\nabla \cdot \delta\boldsymbol{u}\nabla\},$$
$$\nabla \cdot \delta\boldsymbol{v} - \chi\boldsymbol{v}\nabla : \delta\boldsymbol{u}\nabla = 0, \tag{6}$$

which, for the boundary values of the parameter $0 \leqslant \chi \leqslant 1$, reduces to (3) or (5).

The boundary conditions of the examined problem are perturbed similarly, in particular, on the free boundary [8]

$$\boldsymbol{n} \cdot \delta\sigma - \chi\boldsymbol{n} \cdot (\delta\boldsymbol{u}\nabla) \cdot \sigma = \mathbf{0}. \tag{7}$$

In the Eulerian formulation, the perturbation of the current configuration is taken into account in the initial conditions, and in the Lagrangian and intermediate formulations, it is also taken into account in the perturbation-induced change in the local metrics in the equilibrium operator and boundary conditions. In the new formulation, the geometrically nonlinear term is taken into account partially; the reason for this is discussed below.

Stability of Uniaxial Tension. We investigate the stability of uniform uniaxial tension of a superplastic sample with respect to small nonuniform perturbations of the configuration at the current time. This problem is critical for the analysis of the examined approaches to describing motion.

We consider a plane rectangular sample (strip) whose flow in the examined perturbed state is characterized by the relative motion of the ends

$$v_1\Big|_{x=0} = 0, \quad v_1\Big|_{x=l} = v, \quad \sigma_{12}\Big|_{x=0} = \sigma_{12}\Big|_{x=l} = 0$$

with free lateral surfaces:

$$\sigma_{22}\Big|_{y=\pm b/2} = \sigma_{21}\Big|_{y=\pm b/2} = 0$$

(Cartesian coordinates with the x axis directed along the axis of tension is used). In this state, $D_{11} = -D_{22} = v/l \equiv \xi$, $D_{12} = D_{21} = 0$, $\sigma_{11} = \tau$, $\sigma_{22} = \sigma_{12} = \sigma_{21} = 0$, and $\nabla\sigma = 0$.

The perturbed motion is described by the equations following from Eq. (6)

$$m(\delta v_{1,11} - \chi\xi\delta u_{1,11}) + \delta v_{1,22} + \delta v_{2,12} - \chi\xi(\delta u_{1,22} - \delta u_{2,12}) = (\xi/\tau)\,\delta p_{,1},$$
$$m(\delta v_{2,22} + \chi\xi\delta u_{2,22}) + \delta v_{1,21} + \delta v_{2,11} - \chi\xi(\delta u_{1,21} - \delta u_{2,11}) = (\xi/\tau)\,\delta p_{,2}, \tag{8}$$
$$\delta v_{1,1} + \delta v_{2,2} - \chi\xi(\delta u_{1,1} - \delta u_{2,2}) = 0,$$

the conditions on the lateral surfaces following from (7)

$$-\tau\,\delta u_{2,1} + \delta\sigma_{21} = \delta\sigma_{22} = 0 \qquad (y = \pm b) \tag{9}$$

and the conditions on the ends

$$\delta\sigma_{12} = 0 \qquad (x = 0, l) \tag{10}$$

at a constant tensile force. Conditions (10) are satisfied by perturbations in the form

$$\delta u_1 = \varphi(ay)\sin(ax)\,\mathrm{e}^{\lambda\xi t}, \qquad \delta u_2 = \phi(ay)\cos(ax)\,\mathrm{e}^{\lambda\xi t} \tag{11}$$

at $a = \pi n/l$, where n is an integer. In (11) it is taken into account that monochromatic perturbations along the x axis are more dangerous than arbitrary perturbations [9]. Using (4) and (11), from (8) we obtain the equation

$$\phi^{\mathrm{IV}} - 2(2m-1)\phi^{\mathrm{II}} + \phi = 0 \tag{12}$$

(the derivatives are taken with respect to the variable y), which coincides with the equilibrium equation for the streamfunction of the perturbed rate in [7]. From a phenomenological point of view, the coefficient at ϕ^{II} in (12) differs from the same coefficient in [7]. For $m > 0$, the system is an elliptic system, and for $m \leqslant 0$, it is a hyperbolic system. Below, we consider only the first case although the second case, related to the rate softening under dynamic superplasticity is also of interest but it requires a different formulation of the problem. The method of integrating Eq. (12) subject to boundary conditions (9) is described in detail in [5]. We only give one intermediate result — the boundary conditions in terms of the function ϕ:

$$\phi^{\mathrm{III}} + (1-4m)\phi^{\mathrm{I}} = \phi - (\lambda+\chi)(\phi^{\mathrm{II}} + \phi) = 0 \qquad (y = \pm b/2). \tag{13}$$

It should be noted that, in problem (12), (13), the nonEulerian description of motion (involving the preservation of local topology) is concentrated in the term with the factor $\lambda+\chi$ in the condition for the tangential stress on the free boundary, which is characteristic only of the given problem and not of the method proposed. From the equations of the problem we obtain the dependence $\lambda(k)$, where $k = \pi nb/l$:

$$\lambda = \frac{2\sin(k\sqrt{m-1})}{m\sin(k\sqrt{m-1}) \pm \sqrt{m(m-1)}\sinh(k\sqrt{m})} - \chi \qquad (0 < m < 1); \tag{14}$$

$$\lambda = \frac{2\sinh(k\sqrt{m-1})}{m\sinh(k\sqrt{m-1}) \pm \sqrt{m(m-1)}\sinh(k\sqrt{m})} - \chi \qquad (m \geqslant 1). \tag{15}$$

In (14) and (15), the plus and minus signs correspond to the even and odd functions ϕ, respectively, which specifies symmetric or antisymmetric perturbation modes with respect to the tension axis. For the symmetric mode, the function ϕ in the form

$$\phi \sim \cos(\bar\gamma h)\sin(\gamma t) + \cos(\gamma h)\sin(\bar\gamma t), \qquad \gamma = \sqrt{1-m} + i\sqrt{m} \tag{16}$$

is specified according to the range of the parameter m. The function φ is found using the expressions for ϕ and λ (14)–(16) and the relation

$$\varphi = -\frac{\lambda+\chi}{\lambda-\chi}\,a^{-1}\phi',$$

following from the equation of incompressibility of system (8) in view of (11). It is easy to see that solution (11) guarantees that the applied tensile force is constant at the ends of the strip in the integral sense.

In the case $\chi = 1$, the passage to the limit as $a \to 0$ for the symmetric mode corresponds to the uniform perturbation considered in [2]; Eqs. (14) or (15) imply the known result

$$\lambda = (1-m)/m.$$

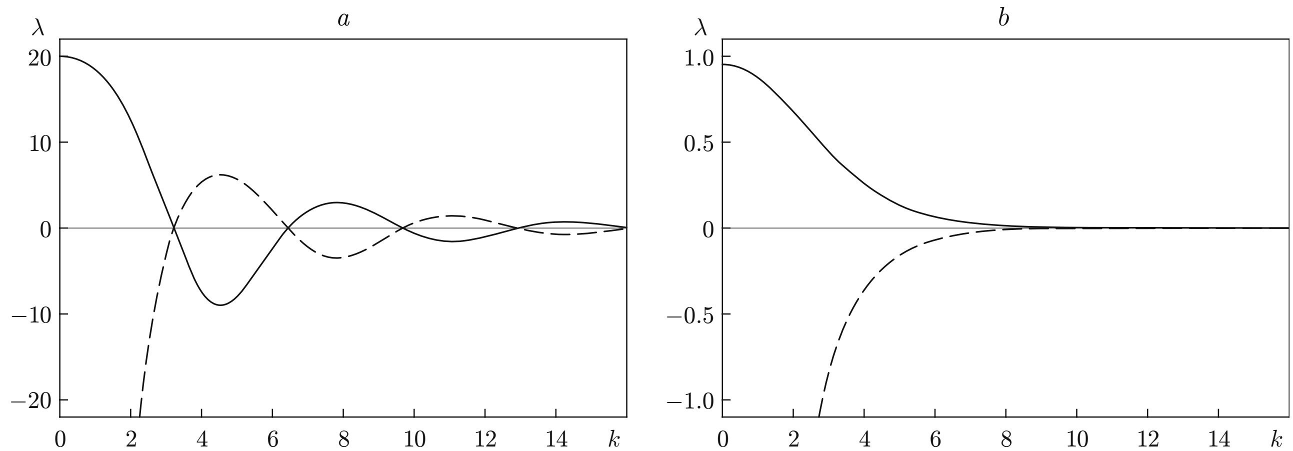

Fig. 2. Dependences $\lambda(k)$ for $\chi = 0$ and $m = 0.05$ (a) and 1.05 (b); the solid curves refer to a symmetric perturbation and the dashed curves refer to an antisymmetric perturbation.

Analysis of Stability Conditions. Figure 2 gives curves of λ [perturbation growth (decay) rates normalized to ξ] versus k [ratios of the strip width b to half the perturbation wavelength along the strip $l/(\pi n)$] for $\chi = 0$.

It should be noted that the symmetric perturbation mode (necks) is more dangerous than the antisymmetric one (wave); therefore, we further consider the symmetric mode. It is obvious that in uniform tension of a strip with a constant rate v of the relative displacement of its ends, the rate deformation ξ decreases in inverse proportion to the sample length l. In this case, ξ falls in the superplasticity range, in which the degree of sensitivity to the rate deformation m reaches the maximum values (see Fig. 1). Each of the configurations the sample takes during uniform tension is independently probed for stability by perturbation by monochromatic modes having different numbers of half-waves and symmetric about the tension axis, and by recording the growth or decay rate of the indicated perturbation. The result of this probing is presented in Fig. 2. Motion to the left along the horizontal axis can be interpreted as a reduction in the number of half-waves of the perturbation imposed on the fixed configuration, or as an elongation of the strip perturbed by a mode with a fixed number of half-waves. For $m < 1$, the strip in tension is periodically in states of stability and instability with respect to perturbations with an arbitrary fixed number of half-waves until, at some critical elongation, it becomes unstable with respect to this perturbation mode. As the parameter χ increases, the curves in Fig. 2 move downward for the quantity χ $(0 \leqslant \chi \leqslant 1)$. Therefore, for $m < 1$ with increasing χ, instability occurs at later times with no qualitative changes occurring in this case. For $\chi = 0$ with increasing m $(m < 1)$, the value of $\lambda(0)$ decreases to reach the value $\lambda = 1$ for $m = 1$. In this limiting state also with increasing χ, instability occurs at later times, and for $\chi = 1$, it does not occur at all. This situation is similar to that examined in [2]. For $m > 1$, there is no alternation of the state of stability and instability with respect to the fixed perturbation mode with elongation of the strip, and the time of onset of instability depends on χ. In this case, since the maximum value of $\lambda = \lambda(0) < 1$ (which decreases with increasing m for $m > 1$), there exists the maximum value χ $(0 \leqslant \chi \leqslant 1)$ for which there is no state of instability. We note that, no matter whether the value of m is higher than unity or not, unstable states exist in the range of the parameters k and χ (or n, l/b, and χ).

The criterion for choosing a method for describing motion, in particular, the values of χ, should be experiment.

Relations (14) and (15) imply that the flow stability is determined only by the material parameter m, which is the current tangential modulus of the curve $\ln\tau(\ln\xi)$, which is characteristic of viscous media [7].

We note that the stability conditions for uniform tension of the sample are excessively stringent; in addition, it is necessary to analyze the stable nonuniform flow modes accompanied by the formation and freezing of single necks [3].

Discussion of the Proposed Method of Equation Perturbation. In studies of the deformation (flow) processes accompanied by a significant distortion of the Lagrangian coordinate grid (local topology change), an effective approach to describing motion is required. Among such processes, in our opinion, are the superplastic deformation modes of polycrystalline media with relative sliding of grains and a change of neighbors [1]. It should be noted that the proposed method of perturbing the equations of motion partially takes into account the continual change in local topology.

The integrability conditions for the current configuration in Euclidean space were obtained in [10]:

$$F \wedge \nabla_0 = \mathbf{0}, \qquad \dot{F} - L \cdot F = 0. \tag{17}$$

Here the fields F and L are given by the differential relations

$$d\hat{\boldsymbol{x}} = F \cdot d\boldsymbol{x}, \qquad d\boldsymbol{v} = L \cdot d\hat{\boldsymbol{x}},$$

where $d\boldsymbol{x}$ and $d\hat{\boldsymbol{x}}$ are infinitesimal material segments in the reference and current configurations, respectively, $d\boldsymbol{v}$ is the relative rate of the segment ends $d\hat{\boldsymbol{x}}$, and ∇_0 is the nabla operator in the reference configuration. The fact that (17) is not satisfied implies that the presence and development of a certain latent field of defects continually changes the local topology of the continuous medium.

Perturbing relation (17), we obtain

$$\delta F \wedge \nabla_0 = \mathbf{0}, \qquad \delta\dot{F} - \delta L \cdot F - L \cdot \delta F = 0. \tag{18}$$

Conditions (18) are satisfied identically in the Lagrangian description. For the first condition, this can be found using the relation $\delta F = \delta\boldsymbol{u}\nabla_0$, and for the second condition, using the relations $\delta\dot{F} = \delta\boldsymbol{v}\nabla_0$ and $\delta L = \delta\boldsymbol{v}\nabla - L\cdot\delta\boldsymbol{u}\nabla$ and $\nabla_0 = \nabla\cdot F$. Within the framework of the proposed description for $0 \leqslant \chi < 1$, the formula $\delta L = \delta\boldsymbol{v}\nabla - \chi L\cdot\delta\boldsymbol{u}\nabla$ changes, resulting in the appearance of the residual $(1-\chi)L\cdot\delta\boldsymbol{u}\nabla$ in the second relation in (18), which is a source of local topology changes, whose power is maximal in the Eulerian description ($\chi = 0$). For $\chi = 1$ in the case of uniform uniaxial tension, the second conditions in (17) and (18) become the identities

$$\frac{\dot{l}}{l_0} - \frac{\dot{l}}{l}\,\frac{l}{l_0} = 0, \qquad \frac{\delta\dot{l}}{l_0} - \Big(\frac{\delta\dot{l}}{l} - \frac{\dot{l}\,\delta l}{l^2}\Big)\frac{l}{l_0} - \frac{\dot{l}}{l}\,\frac{\delta l}{l_0} = 0,$$

and for $\chi \neq 1$,

$$\frac{\delta\dot{l}}{l_0} - \Big(\frac{\delta\dot{l}}{l} - \chi\,\frac{\dot{l}\,\delta l}{l^2}\Big)\frac{l}{l_0} - \frac{\dot{l}}{l}\,\frac{\delta l}{l_0} = (1-\chi)\,\frac{\dot{l}\,\delta l}{l^2} \neq 0,$$

which supports the detected violation of the consistency of the kinematic fields. Thus, the given phenomenological material parameter allows one to take into account the new property of the medium — the degree of preservation of local order — irrespective of the constitutive equations used.

This source of local topology changes has a gradient form and generates the latent vector field $(1-\chi)L\cdot\delta\boldsymbol{u}$ [11]. To elucidate its meaning, we write the equations of the trajectory and the streamline through the same point in the current configuration:

$$\frac{d\hat{\boldsymbol{x}}}{dt} = \boldsymbol{v}(\hat{\boldsymbol{x}}, t), \qquad \frac{d\hat{\boldsymbol{x}}'}{ds} = \boldsymbol{v}(\hat{\boldsymbol{x}}', t). \tag{19}$$

In the streamline equation, we set the parameter s to be coincident with t, by fixing t on the right side of this equation. Then, Eq. (19) implies the following relation for the instantaneous rate of divergence of the trajectories and streamlines at the given point:

$$\frac{d\,\delta\hat{\boldsymbol{x}}}{dt} = \boldsymbol{v}\nabla\cdot\delta\hat{\boldsymbol{x}}. \tag{20}$$

Because $\delta\hat{\boldsymbol{x}} = \delta\boldsymbol{u}$ in (20), for $\chi = 0$, the right side of the relation is a source of local topology changes, and for $\chi = 1$, this source is absent. Taking into account that, in the Lagrangian approach, perturbation of the motion of a material point occurs along the trajectory of this point, we conclude that, in the Eulerian approach, the same perturbation moves a material point along the streamline through the point of space occupied at the moment by the material point considered. The meaning of the latent vector field $(1-\chi)L\cdot\delta\boldsymbol{u}$ now becomes clear in the intermediate cases $\chi \neq 0$ and $\chi \neq 1$, too. Thus, the meaning of the perturbation of the current configuration was established using various approaches to describing motion.

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